Indian Statistical Institute, Bangalore

B. Math. First Year, Second Semester

Probability Theory-II Final Examination

Duration: 3 hours Maximum marks: 100 Date : 09-05-2012

1. Let D, E be outcomes of two independent throws of a die. Suppose $M = \text{Min}\{D, E\}$ and $N = \text{Max}\{D, E\}$. Find joint distribution and marginals of M, N. Compute expectation and variances of M, N. [10]

2. Suppose Y_1, Y_2 are independent identically distributed random variables with density g given by $g(y) = \frac{1}{2\lambda}e^{-\frac{|y|}{\lambda}}$ for all real y, with some fixed $\lambda > 0$. Find the distribution function and a density function for $Y_1 + Y_2$. [15]

3. Let V, W be independent, identically distributed random variables with density given by
$$h(t) = \begin{cases} e^{-t} & \text{for } t > 0; \\ 0 & \text{otherwise.} \end{cases}$$
Obtain densities for $V + W$ and $\frac{V}{W}$ and prove or disprove the statement: $V + W$ and $\frac{V}{W}$ are independent. [20]

4. Let S_1, S_2 be i.i.d. random variables with common density:
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \quad -\infty < x < \infty.$$
Take $T_1 = 2S_1 + 5S_2$ and $T_2 = -5S_1 + 2S_2$. Find the joint density of T_1, T_2. Find the marginal density of T_1. [20]

5. Suppose $\{X_n\}_{n\geq 1}$ is a sequence of random variables converging in distribution to a random variable X. Show that (i) X_n^2 converges to X^2 in distribution as n tends to infinity. (ii) Show that $-X_n$ converges to $-X$ in distribution as n tends to infinity. [20]

6. State and prove Weak Law of Large Numbers (WLLN) for i.i.d. sequences of random variables with finite variance. [20]